301. Two white counters and two black counters are placed in a bag. Two counters are then drawn out. Find the probability of drawing out
   (a) two white counters,
   (b) one of each colour.

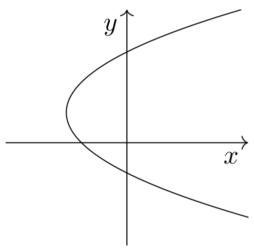
302. To solve the equation $x^3 = x$, a student writes the following: "Dividing through by x gives $x^2 = 1$, so $x = \pm 1$." Explain what is wrong with this logic.

303. Solve $|3x - 1| = |2 - 5x|$.

304. "The curves $y = x^2$ and $y + x^2 + 4x + 2 = 0$ are tangent to one another." True or false?

305. The parabola $y = ax^2 + bx + c$ passes through the origin. Explain whether any of the constants a, b, c can be determined from this information.

306. State, with a reason, which of these equations could possibly be that of the sketched graph.



   (a) $x = (y - 1)^2 - 2$,
   (b) $x = (y + 1)^2 - 2$,
   (c) $x = (y - 1)^2 + 2$.

307. Triangle ABC has $|AB| = 2\sqrt{3}$, $|BC| = 2$, and $\angle CAB = 30°$. Find all possible values of $\angle BCA$.

308. (a) Show that $x^2 + 1 = x$ has no real roots.
   (b) Hence, show that the instruction $x \mapsto x^2 + 1$ maps all real numbers to numbers larger than themselves.

309. The equations $x + 2y = a$, $3x - 4y = 6$ are satisfied simultaneously by $x = 4$, $y = b$. Find a and b.

310. A function f is defined over the real numbers $\mathbb{R}$ by $\mathrm{f} : x \mapsto 4(x - 2)^3 - 1$. Find an algebraic expression for $\mathrm{f}^{-1}(x)$.

311. Two of the following statements are true; the other is not. Prove the two and disprove the other.
   (a) $(x - 2)(x^2 - 4x + 3) = 0 \implies x = 2$,
   (b) $(x - 2)(x^2 - 4x + 4) = 0 \implies x = 2$,
   (c) $(x - 2)(x^2 - 4x + 5) = 0 \implies x = 2$.

312. By substituting the variable $z = x^3$, or otherwise, solve the equation $x^6 - 9x^3 + 8 = 0$.

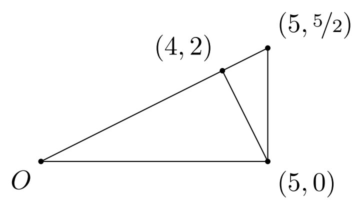
313. Find and correct the error in the following:

$$\begin{aligned} & x^4 - x^2 = 0 \\ \implies & x^2 - 1 = 0 \\ \implies & x = \pm 1. \end{aligned}$$

314. Verify that the function $\mathrm{f}(x) = 4x^{\frac{1}{3}}$ is a solution of the differential equation

$$\mathrm{f}'(x) = \frac{\mathrm{f}(x)}{3x}.$$

315. Show that the triangles below are all similar.



316. Show that $2\displaystyle\int_1^k t - 1 \, dt = (k - 1)^2$.

317. If $y = x - 2$, write $x^2 - 4x$ in simplified terms of y.

318. Two disgruntled elephants are having a shoving match. Neither is giving an inch. Explain how Newton's laws tell you that the magnitudes of the following forces are equal:
   (a) the force of elephant A on elephant B; the force of elephant B on elephant A,
   (b) the frictional force of the ground on elephant A; the force of elephant B on elephant A.

319. A graph is given by $(x^2 + y^2 - 1)(x^2 + y^2 - 4) = 0$.
   (a) Explain why the graph consists of a pair of distinct circles.
   (b) Sketch the graph.

320. The dimensions of a sheet of A4 are defined such that, when the sheet is folded in half to make A5, the shapes are similar. Show that the dimensions of such sheets of paper are in the ratio $1 : \sqrt{2}$.

321. Show that $(3x + 1)^2(x + 2) + 6x + 2$, where $x \in \mathbb{N}$, cannot be prime.

322. (a) Evaluate $6x^3 - 23x^2 + 24x - 7\Big|_{x=\frac{1}{2}}$
   (b) Hence, write down a factor of the expression $6x^3 - 23x^2 + 24x - 7$, giving the name of the theorem you use.

323. Write the following in simplified interval notation:

$$\{x \in \mathbb{R} : x > 0\} \cap \{x \in \mathbb{R} : x \le 3\}.$$

324. Sketch a linear graph $y = \mathrm{f}(x)$ for which

$$\int_{-1}^{1} \mathrm{f}(x)\, dx = 0, \quad \mathrm{f}(-1) = 1.$$

325. There are 4 paths from point A to point B. These paths do not intersect en route. Determine the number of distinct return trips from A back to A via B, if the paths used out and back

(a) may be the same,

(b) must be different.

326. Solve the following equation:

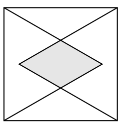
$$\frac{1 + {}^{1}\!/_{x}}{1 - {}^{1}\!/_{x}} = x.$$

327. Two dogs are pulling in opposite directions on a light toy. The magnitudes of the forces applied by the dogs on the toy are D_1 and D_2 N. Explain carefully, with reference to Newton's laws, why the difference $D_1 - D_2$ must be negligible.

328. The scores X and Y on two tests, each out of forty marks, are to be combined, with equal weighting, into a single mark M, which is out of a hundred. Determine a formula for M in terms of X and Y.

329. Show that $\left(1 + \sqrt{3}\right)^5 = 76 + 44\sqrt{3}$.

330. Two equilateral triangles are drawn inside a square of side length 1.



Show that the area of the shaded region is $\frac{2\sqrt{3}}{3} - 1$.

331. Sketch the graph $\dfrac{x^2}{y^2} = 1$.

332. A particle has acceleration of magnitude 35 ms^{-2} in the direction of $-6\mathbf{i} + 8\mathbf{j}$. Find the component of its acceleration in the $\mathbf{j}$ direction.

333. A function f is such that the indefinite integral of f is quadratic. Describe the function f.

334. A circle, in the positive quadrant, has radius 1 and is tangent to both axes.

(a) Write down the equation of the circle.

(b) The circle is stretched by scale factor 2 in the x direction. Write down the equation of the ellipse formed.

(c) The ellipse is then reflected in the line $y = x$. Write down the equation of the new ellipse.

335. An arithmetic sequence has first term 1, last term 15 and sum 64. Find the common difference.

336. Polygon $OABC$ has vertices at $(5, 1), (1, 5), (6, 6)$.

(a) Show that the polygon is a rhombus.

(b) Show that the area of the rhombus is 24.

337. Show that $\mathrm{f}(x) = 10 + 24x^2 - 16x^4$, when defined over $\mathbb{R}$, has maximum value 19.

338. Explain why, if three events A, B, C are pairwise mutually exclusive, then

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C).$$

339. The sequence of odd numbers can be thought of as an AP with common difference 2. Use this fact to find the sum of the first 100 odd numbers.

340. An object has exactly three forces acting on it, whose magnitudes are 2, 5, 8 Newtons. Show that the object cannot be in equilibrium.

341. Evaluate the following:

(a) $\left[2^x + 3^x\right]_0^1$

(b) $\left[\log_2 x + \log_4 x\right]_1^2$

342. A chord, which is tangent to $x^2 + y^2 = 1$, is drawn to $x^2 + y^2 = 4$. Determine its exact length.

343. $2n$ fair coins is tossed, and the number of heads is denoted X. X has the binomial distribution $X \sim B\left(2n, \frac{1}{2}\right)$. Prove that, for all k,

$$\mathbb{P}(X = n + k) = \mathbb{P}(X = n - k).$$

344. A function f, defined over $\mathbb{R}$, has $\mathrm{f}(0) = 3$ and $\mathrm{f}'(0) = 2$. Also, $\mathrm{f}''(x) = 4$ for all $x \in \mathbb{R}$.

(a) Show that $\mathrm{f}'(x) = 4x + 2$.

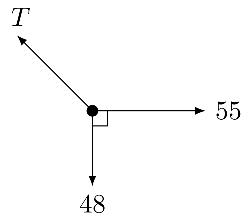
(b) Find $\mathrm{f}(x)$.

(c) Sketch $y = \mathrm{f}(x)$.

345. Using the factor theorem, show that $(x^2 - 1)$ is a factor of $x^3 - 4x^2 - x + 4$.

346. The radian is defined so that arc length l is given by $l = r\theta$, where θ is the angle subtended at the centre of the circle. Prove, from this definition, that sector area A is given by $A = \frac{1}{2}r^2\theta$.

347. Three forces, with magnitudes given in Newtons, hold a mass in equilibrium as depicted below. The 55 N and 48 N forces are perpendicular.



Find T.

348. Find $\displaystyle\int \frac{(2x+1)(2x-1)}{x^2}\,dx$.

349. A hand of three cards is dealt from a standard deck. Find the probability that the hand contains no face cards (Jack, Queen, King).

350. Consider a quadratic graph $y = x^2 + px + q$.

(a) Show that this can be written in the form

$$y = \left(x + \tfrac{1}{2}p\right)^2 + q - \tfrac{1}{4}p^2.$$

(b) Explain how you know that the graph has a minimum at the point

$$\left(-\tfrac{1}{2}p, q - \tfrac{1}{4}p^2\right).$$

351. A function h, for which $\mathrm{h}(0) = 1$ and $\mathrm{h}(1) = 4$, has constant first derivative. Find $\mathrm{h}(3)$.

352. State whether the following functions have a sign change at $x = a$.

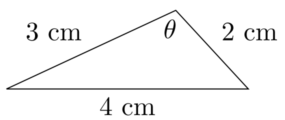
(a) $\mathrm{f} : x \mapsto (x - a + 1)(x - a)$,

(b) $\mathrm{f} : x \mapsto (x - a + 1)(x - a)^2$,

(c) $\mathrm{f} : x \mapsto (x - a + 1)(x - a)^3$.

353. Solve the equation $x - \left(\sqrt{x} - 2\right)^2 = 1$.

354. Triangle ABC has sides $2, 3, 4$ cm.



(a) Use the cosine rule to find $\cos\theta$.

(b) Hence, show that $\sin\theta = \frac{1}{4}\sqrt{15}$.

(c) Using $A = \frac{1}{2}ab\sin C$, determine the area of the triangle, giving your answer exactly.

355. Solve the equation $|y^2 - 2| = 5$.

356. Write down the number of ways of rearranging:

(a) PQRS,

(b) PPQR.

357. A function f has $\mathrm{f}''(x) = 6\sqrt{x}$. The graph $y = \mathrm{f}(x)$ has gradient 5 as it crosses the y axis. Find $\mathrm{f}'(x)$.

358. Solve for x in the equation $ax^2 + bx = 0$, where $a, b \in \mathbb{R}$ and $a \neq 0$.

359. State whether the following hold:

(a) $x^2 = y^2 \implies x = y$,

(b) $x^2 = y^2 \impliedby x = y$,

(c) $x^3 = y^3 \implies x = y$,

(d) $x^3 = y^3 \impliedby x = y$.

360. Convert the following into units of radians, giving your answers in terms of π:

(a) $30°$,

(b) Four and a half revolutions.

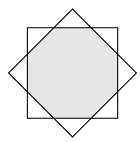
361. State, with a reason, whether the line $ax + by = c$ intersects the following lines:

(a) $ay - bx = c + 1$,

(b) $ax + by = c + 1$,

(c) $ax - by = c + 1$.

362. Two squares of side length 1 are overlaid to form the symmetrical pattern below.



Show that the area of the octagon is $\sqrt{8} - 2$.

363. Two hikers leave camp simultaneously. A walks on bearing 160° at 3 mph; B walks on bearing 250° at 4 mph. Determine the time it takes for them to separate by five miles.

364. Either prove or disprove the following implications:

(a) $x + y > 1 \implies x^2 + y^2 > 1$,

(b) $x^2 + y^2 > 1 \implies x + y > 1$.

365. Find the length of the space diagonal of a cuboid measuring $4 \times 4 \times 7$.

366. Exactly three forces $\mathbf{F}_1 = 10\mathbf{i}$ N, $\mathbf{F}_2 = 20\mathbf{j}$ N and $\mathbf{F}_3$ act on a particle of mass 10 kg. The particle accelerates from rest in the direction of the vector $\mathbf{i} + \mathbf{j}$. After t seconds, the object has covered a distance of $s = 2\sqrt{2}t^2$ metres. Find the force $\mathbf{F}_3$.

367. The parabola $y = (x-a)^2 + b$ is rotated 180° around its vertex. Write down the equation of the new parabola.

368. Find $\frac{dy}{dx}$ when

(a) $y = x(x-3)(x+4)$,
(b) $xy = x^2 + 2$.

369. Determine the two integer values of x which satisfy the inequality $x^2 + 5x + 5 < 0$.

370. A circle of radius 1 is placed at random with its centre somewhere (equally likely to be anywhere) inside a square of side length 3. Determine the probability that circle and square intersect.

371. The following identity is proposed:

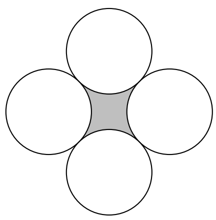
$$x^2 \equiv A(x+1)^2 + B(x-1)^2.$$

By equating coefficients, or otherwise, show that no such constants A, B can be found.

372. Find all possible values of k such that $(x-1)$ is a factor of $k^2x^3 - 12x^2 + 2k^2$.

373. Prove that, if two functions f and g have the same first derivative $f'(x) = g'(x)$, then the curves $y = f(x)$ and $y = g(x)$ are a constant distance apart in the y direction.

374. Four circles of radius 1 are arranged symmetrically, such that each circle is tangent to two of the others and their centres lie at the vertices of a square.



Find the exact area of the shaded region.

375. Prove formally that the difference between two odd numbers is even.

376. Show that $f(x) = 7x^2 - 9x + 12$ produces outputs that are always greater than 9.

377. True or false?

(a) $\sqrt{x} = \sqrt{y} \iff x = y$,
(b) $\sqrt[3]{x} = \sqrt[3]{y} \iff x = y$,
(c) $\sqrt[4]{x} = \sqrt[4]{y} \iff x = y$.

378. Shade the regions of the plane for which $xy \geq 0$.

379. The velocity of a moving particle, at time t, is given by $v = 4t + 6$, and the average velocity over the first k seconds of motion is 20 ms^{-1}.

(a) Show that $\dfrac{1}{k}\Big[2t^2 + 6t\Big]_0^k = 20$.
(b) Solve for k.

380. If $f(x) = 3x^2 - 2x + 1$, find $\displaystyle\int f'(x)\,dx$.

381. Beginning with the first Pythagorean trig identity $\sin^2\theta + \cos^2\theta \equiv 1$, prove, by enacting a suitable division, that $\tan^2\theta + 1 \equiv \sec^2\theta$.

382. A student is looking for an algebraic slip in his work. Explain the error and correct it.

$$\frac{3x-2}{5x-1} - \frac{x-1}{5x-1} = 1$$
$$\implies 3x - 2 - x - 1 = 5x - 1.$$

383. A sequence is defined by $u_n = 100 - 85n + 3n^2$. Find the minimum value of the sequence.

384. Two forces $\mathbf{F}$ and $\mathbf{G}$ act on an object, which remains in equilibrium. In terms of constants $a, b \in \mathbb{R}$, the forces are

$$\mathbf{F} = 4a\mathbf{i} + b\mathbf{j},$$
$$\mathbf{G} = (2b+2)\mathbf{i} + (a+3)\mathbf{j}.$$

Find a and b.

385. Prove algebraically that, if three variables a, b and c are linked by $a \propto b$ and $b \propto c$, then $a \propto c$.

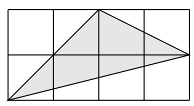
386. A function g has first derivative $g'(x) = 2x + 3$. Evaluate $g(2) - g(1)$.

387. Two cards are dealt from a standard deck. State which, if either, of the following events has the greater probability:

1. two jacks,
2. a king and a queen.

388. Describe the transformation(s) which map graph $y = f(x)$ onto graph $y = 3f(x) + 2$.

389. The grid shown below consists of unit squares.



Find the area of the shaded region.

390. Solve the simultaneous equations

$$0.1x + 0.3y = 0.13,$$
$$x^2 - 0.6y = 0.37.$$

391. A convex quadrilateral has vertices at $(0,0)$, $(3,8)$, $(9,4)$, and $(10,0)$. Show that its area is 50.

392. Vectors are given as $\mathbf{p} = \mathbf{i} + 2\mathbf{j}$ and $\mathbf{q} = 2\mathbf{i} + \mathbf{j}$. The vector $11\mathbf{i} + 13\mathbf{j}$ is to be written as a linear combination $a\mathbf{p} + b\mathbf{q}$. Find a and b.

393. Position x takes the following values at time t:

t	0	5	10	15
x	4	5	8	13

Show that this position data is consistent with the assumption of constant acceleration.

394. Consider the function $\mathrm{g}(x) = 224x^2 - 649x + 255$.

(a) Show that g has two rational roots.

(b) Hence, factorise $\mathrm{g}(x)$.

395. By listing them explicitly, show that there are ten possible rearrangements of AAABB.

396. The interior angles of a triangle are in arithmetic progression. Give, in radians, the set of possible values for the smallest angle.

397. You are given that the equations $3x - py = 5$ and $6x = 2y + q$ have no simultaneous solutions (x, y).

(a) Explain why the two equations must describe a pair of parallel lines.

(b) Write down the value of p.

(c) Determine all possible values of q.

398. Explain why no quadratic function has range $\mathbb{R}$.

399. By factorising, or otherwise, prove that, for $n \in \mathbb{N}$, $n^2 + n$ is always even.

400. State, with a reason, whether the following claims are true in the Newtonian system:

(a) "An object with mass cannot be weightless."

(b) "An object with weight cannot be massless."

——— END OF 4TH HUNDRED ———